M.Sc. - II (Mathematics) (NEP Pattern) Semester-IV

04NEPMATH04.1 - Elective : Integral Equations

P. Pages : 2 **GUG/S/25/16361**

Time : Three Hours * 3 9 5 5 * Max. Marks : 80

Notes : 1. Solve all **five** questions.
2. Each equation carry equal marks.

UNIT – I

1. a) Show that the function u(x) = 1 is a solution of the Fredholm integral equation. **8**

$$u(x)+\int_0^1 x\left(e^{xt}-1\right)u(t)\,dt=e^x-x.$$

b) Show that the function $u(x)=xe^x$ is a solution of the Volterra integral equation. **8**

$$u(x)=\sin x+2\int_0^x \cos(x-t)\,u(t)\,dt.$$

OR

c) Show that integral equation corresponding to the differential equation $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0$ with initial condition $y(0)=0,\ y'(0)=-1$. **8**

d) Convert the following differential equation into an integral equation $y''+y=0$ with initial conditions $y(0)=y'(0)=0$. **8**

UNIT – II

2. a) Find the eigen values and the corresponding eigen functions of the Homogeneous integral equation $u(x)=\lambda\int_0^1 \sin\pi x\ \cos\pi t\ u(t)\,dt$. **8**

b) Solve $u(x)=x+\lambda\int_0^1 \left(xt^2+tx^2\right)u(t)\,dt$. **8**

OR

c) Find the solution of the integral equation $u(x)=f(x)+\lambda\int_0^{2\pi}(\sin x\ \cos t)\,u(t)\,dt$. **8**

d) Solve $u(x)=1+\int_0^1\left(1+e^{x+t}\right)u(t)\,dt$. **8**

UNIT – III

3. a) State and prove Schwarz Inequality. **8**

b) State and prove Bessel’s Inequality. **8**

OR

c) Solve the following homogeneous integral equation using Schmidt solution **8**

$f(x) = \lambda \int_0^1 e^x e^t f(t)\,dt,$

d) Solve the following symmetric integral equation with the help of Hilbert – Schmidt theorem **8**

$u(x) = 1 + \lambda \int_0^{\pi} \cos(x+t)\,u(t)\,dt.$

UNIT – IV

4. a) Prove that $R(x,t;\lambda)$ be the reciprocal kernel of a Fredholm integral equation **8**

$u(x) = f(x) + \lambda \int_a^b k(x,t)\,u(t)\,dt$ then $R(x,t;\lambda) = k(x,t) + \lambda \int_a^b k(x,s)\,R(s,t;\lambda)\,ds.$

b) Solve the following integral equation $u(x) = \frac{5x}{6} + \frac{1}{2} \int_0^1 xt\;u(t)\,dt.$ **8**

OR

c) Solve $u(x) = f(x) + \frac{1}{2} \int_0^1 e^{x-t} u(t)\,dt.$ **8**

d) Find the solution of the integral equation $u(x) = 1 + x^2 + \int_0^x \frac{1+x^2}{1+t^2}\,u(t)\,dt.$ **8**

5. a) Define the eigen values and eigen function for the integral equation. **4**

b) Define index of the eigen value and discuss the case F(x) = 0 for the Fredholm equation with separable Kernel. **4**

c) Show that eigen value of symmetrical kernel are real. **4**

d) Find $k_2(x,t)$ for Kernel **4**

$k(x,t) = (1+x)(1-t),\ a = -1, b = 0.$

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